$\mathring{\mathcal{S}}$ – Introducing sonification variables

Julian Rohrhuber

1 Abstract

This short paper introduces the notion of *sonification variables*, which help in the field of sonification, in moving beyond analysis of measured data towards the theoretical relations between them. A new symbolism is suggested to make this possible independently of specific disciplines.[1] The introduction of such variables into arbitrary formalisms is described in terms of a general *sonification operator* $\mathring{\mathcal{S}}$.

2 Mixed expressions

Sonification has mostly been centred around the perceptualisation of given data. However important measurement is – just as observation is theory laden,[2] data are theory laden too: observations are embedded in a theoretical context that attempts to explain their inner logic. From practical experience we also know that this inner logic is not confined to the domain science, but also enters the sonification process – after all, sonification should help to understand the domain and prompt new research questions.

In many cases, this theoretical background is made explicit in mathematical or formal terms. Therefore, in the sonification research process, two types of formalisms appear:

---

[1]The ideas in this paper result from an ongoing conversation with Henri Kowalski and from an inspired discussion at the recent Workshop *Science by Ear II* in Graz. Special thanks to the discussants and to Kathi Vogt, Till Bovermann and Alberto de Campo for the specific suggestions.

[2]There persists a long debate sround the theory ladenness of observation and measurement. An important turning point is the demonstration that synthetic and analytic truths cannot be completely separated (Quine (1951)). See also: Heidelberger (2003).

- The target domain contributes symbolic expressions peculiar to the field, be it in the form of data formats or mathematical expressions, and
- the method of sonification implies formalisms for signal processing, acoustics, and in particular some programming language.

Under the assumption that both observation and perceptualisation are inherently theory laden, the theoretical context of data makes an essential part of their significance.

Therefore, the argument and starting point of the present paper is that *formalisms used in the domain science should be included into the investigation of sonification proper*. Usually, data formats play the role of such intermediaries, and much time is spent in sonification trying to understand their significance. In sonification research it can be quite challenging to communicate between the different disciplines involved, and especially the formalisms are often not understood equally well by everybody in the group. An effort to improve this situation must obviously remain somewhat incomplete, and depends on each situation and combination of disciplines. The step proposed here however, is to move the sonic away from being an end point ('display'), and interlace it with the target domain's specific way of reasoning. One way to achieve this is to unify sonification specific and domain specific formalisms, communicating and reasoning with *mixed expressions*. Such a mixture seem promising because it avoids a preconceived interface between exploration and established knowledge and provokes a reconsideration of assumptions, in the domain as well as in sonification.

3 Sonification time

A good starting point for finding such a form of mixed expression is the sonification in physics. Since we may express a sound signal in terms of its physical properties, it is relatively easy to express a given mathematical formalism for a physical phenomenon in terms of a wave form, a function of pressure or displacement over time.

Let's think of a really simple example from classical mechanics: a quickly and uniformly rotating body with radius r and angular velocity ϕ can be interpreted directly as a sound wave by considering only one component of its movement, in terms of its displacement (commonly expressed by the variable y):

$$y(t) = r \sin(\phi t) \tag{1}$$

Obviously, the time variable t has a very specific role in this expression. Two aspects are superposed in it: the *domain time* (the time passing during rotation) and the *sonification time* (the time passing while listening to the sonification). As it turns out, we have thereby tentatively assumed that the physical time and the time of sonification are the same. This assumption is only justified however as a boundary condition of sonification, a kind of 'immediate mapping', in which one parameter is interpreted as physical displacement.

Let's assume that the body rotates slowly, so that the frequency of the displacement is below audible range. To sonify this process, we may either

- speed it up in order to listen (analogous to data *audification*), or
- include it in a more complex sonification process, for instance interpret the displacement as a change of an audible signal instead of the signal itself (*parameter mapping*), or use its trajectory (in this case a circle) as a physical model (*model based sonification*).

The first simple case serves well enough as an example. It can be formalised by explicitly introducing a temporal scaling factor a (which can be chosen to shift the frequency into audible range), and a second scaling factor for the resulting amplitude b (which can be chosen to shift the acoustic amplitude into audible range):

$$y(t) = rb\sin(a\phi t). \tag{2}$$

The oscillation frequency $a\phi$ thus immediately specifies an audible signal. The factors a and b express a constant modification of the body's rotational movement. It seems however that this is not really what was intended in the first place: not the body should rotate quicker, but the sonification should allow a different viewpoint on the same physical process.

This minimal example shows that mixed expressions can be misleading. Sonification time is an entirely free parameter, while physical time isn't necessarily one.[3] Instead of modifying the properties of our domain, we should therefore rather explicitly separate physical time and sonification time.

[3]Obviously, another perspective is possible here: The acceleration factor may also taken as accelerating time itself instead of movement $((a\phi)t)$, so that we also read $((at)\phi)$. A physical interpretation of time as a free variable could be taken into account by interpreting it as a Lorentz transformed reference frame. This may be stretching the analogy too far.

4 Sonification variables

Wherever they coexist, it is necessary to explicitly mark the difference between the two conceptual reference frames of sonification and of the domain phenomenon. In many sonification approaches, this is achieved by first generating data within the domain formalism and only afterwards perceptualising those data with the help of a sonification formalism. Within the proposed mixed expressions, this can be done conveniently by superscribing variables that belong to the context of sonification by a *ring*.[4] Sonification time is therefore distinguished as $\mathring{t}$ from the domain time variable t.

While it was our initial point of distinction, time is not the only dimension that may come in as a property peculiar to sonification. Most importantly, together with *sonification time*, the *audio signal* itself should be similarly marked. Following convention, the sonification signal would be called $\mathring{y}$.

The above example can now be rewritten as

$$\mathring{y}(\mathring{t}) = r \sin(\phi \mathring{t}), \tag{3}$$

where $\mathring{t} = at$ and $\mathring{y} = by$, separating the above scaling factors and making the relation between sonification and domain explicit.

The true benefit of this marking becomes clear only in more complicated terms, especially when both domain time and sonification time occur together. A side effect is that it also allows for expressions where the signal is not in normal form $\mathring{y} = \ldots$, but $\mathring{y}$ may occur anywhere in the formula, before it is resolved. Note also that not all formulas have a time variable, so that by introducing an explicit name $\mathring{t}$, we can be sure that we do not think of the domain as changing in time, but as listened to in time.

Is this all we need? Sonification may be taken at least as a sound signal, being a function of time. By conclusion, for a minimal formalisation of sonification, at least two specific variables are needed, which we have named $\mathring{t}$ and $\mathring{y}$. Depending on the conventions of the domain, other names may be chosen to avoid ambiguities. This is a minimal set: Other variables may come in when we choose another base for describing the sound signal.

[4] In LaTeX, the little ring is written as \mathring{...}

5 The sonification operator

Note that in difference to the variables that appear in the initial formalism, the sonification variables have no strict interpretation in the domain. Thus, one further conclusion of this simplification can be drawn. Let's reconsider: what we have done by introducing a special marking into the given domain formalism was effectively to translate one formalism A (a function, relation or other term) into another one (which we may call $\mathring{A}$ now). This mapping introduces at least two variables, the *sonification time* and the *sonification signal*. We write a formalisation of sonification as an intervention in a domain as:

$$\mathring{\mathcal{S}} = A\langle \xi \rangle \mapsto \mathring{A}\langle \xi, \mathring{t}, \mathring{y} \rangle \tag{4}$$

where ξ is the set of domain variables $x_0, x_1, \ldots x_n$, which may specify the sonification, but belong to the initial formalism itself and usually have an interpretation in the domain. The sonification formalism $\mathring{A}$ describes an implicit or explicit relation between $\mathring{y}$ and $\mathring{t}$, depending on a subset of the parameters ξ.

In terms of its dimensions, the sonification operator $\mathring{\mathcal{S}}$ may be considered a map of the extension of A (taken as an arbitrary set $\mathbb{A}$ here) to an n-dimensional continuous audio signal $\mathbb{R}^n$, assuming a real-valued time domain.

$$\mathring{\mathcal{S}} : \mathbb{A} \times \mathbb{R} \mapsto \mathbb{R}^n \tag{5}$$

Accordingly, a very basic but typical case could be the interpretation of a series of natural numbers as pitch levels of a stereo sound source, which would mean that $\mathring{\mathcal{S}} : \mathbb{N} \times \mathbb{R} \mapsto \mathbb{R}^2$. Another could be a sonification of the differential equations of the three body problem in celestial mechanics[5] to a mono sound signal, where $\mathring{\mathcal{S}} : \mathbb{R}^{18} \times \mathbb{R} \mapsto \mathbb{R}$. On a purely implementation level on the other hand, sonification is interpreted as finite sets, or maximally as countably infinite sets: $\mathring{\mathcal{S}} : \mathbb{N} \times \mathbb{N} \mapsto \mathbb{N}$

6 Towards operator based sonification

Should sonification be included in the reasoning about the domain it is involved in, it is very useful to introduce *mixed expressions* that intertwine

[5]The problem describing the motion of three celestial bodies, requiring three times six degrees of freedom. For an ecellent historically contextualised discussion see Galison (2004).

sonification method and domain formalism. We have introduced a sonification operator $\mathring{\mathcal{S}}$, which explicitly translates a domain formalism into a description of an audible process. This introduces *sonification variables*, which we mark by a ring to avoid ambiguities.

The specific type of sonification that involves not only data, but domain formalisms, may be named *operator based sonification*; it can be considered one extreme of a wide and entangled spectrum between sonification of measurement and sonification of theory. It would seem like operator based sonification approaches can be conveniently combined with other sonification methods. For instance, we may interpret a formal term as a multi-dimensional data space and then use it in a physical mechanical model (*model based sonification*[6]). Also, measured data can be inserted in the sonification formalism which can be used in a black-box *parameter mapping*.

However, all this is but a first step. Two main difficulties are in view: finding ways to write more advanced sound algorithms in a way making them readable in other domains, and finding interesting domain formalisms for sonification. The latter may require to move beyond a basic physical description[7]

Already in simple cases, using sonification variables helps at least to distinguish sonification time from domain time; it clarifies the difference within sonification methods between those aspects which are needed for accessing and preparation and those related directly to listening.

[6] Hermann and Ritter (2004).

[7] Many formalisations of sound and music have been proposed, but the task is not trivial if one accepts the necessity to account for properties normally taken as secondary qualities, timbre being the most prominent. For some discussion, see Kaper and Tipei (1999); Xenakis (2001). In general, higher level sound programming languages may be considered as such formalisations, however lacking the conventions of scientific scripts.

7 Appendix: examples for mixed expressions and their interpretation in SuperCollider

7.1 Additive Synthesis

In its simplest form, additive synthesis combines multiple layers of pure sine waves, each of which have a specific frequency, phase, and amplitude. A special case for equally spaced frequency bands is Fourier analysis. Here is an example where they are spaced according to a square root function.

```
(
play {
  var n = 5, m = 17;
  (1/(m-n)) * (n..m).sum { |i| 1/(i + 1) * SinOsc.ar(300 * sqrt(i)) }
}
)
```

$$\mathring{y}(\mathring{t}) = \frac{1}{m-n} \sum_{i=n}^{m} \frac{1}{i+1} \sin(\sqrt{i}\mathring{t})$$

where $n = 5, m = 17, \mathring{t} = t\frac{300}{2\pi}$

7.2 Example: Frequency and Phase Modulation Synthesis (FM/PM)

Modulating the frequency (or phase) of one periodic function (the carrier) with another (the modulator) results in a rich and well known sound spectrum. Here a slightly more complex example, where the modulator signal in turn is modulated by the resulting signal.

```
(
Ndef(\y, { | vmod=40, vcarr=300, i=10, z=3.7 |
   SinOsc.ar(vcarr, i * SinOsc.ar(vmod * (Ndef.ar(\y) + z))) * 0.1
}).play
)
```

$$\mathring{y} = a \sin(\nu_c \mathring{t} + I \sin(\nu_m \mathring{t}(z + \mathring{y}))),$$

where $\mathring{t} = t\frac{1}{2\pi}$, $\nu_c = 300$, $\nu_m = 40$, $I = 10$, $a = 0.1$, and $z = 3.7$.

7.3 Example: Waveshaping Synthesis

Waveshaping synthesis is implemented by using a signal x as a dynamic index into a wave form z, usually represented as a buffer. Mathematically, waveshaping may simply be expressed as function composition.

```
(
play {
   var f = { |x|  (0.3 * (x * x)) - (0.8 * (x * x * x)) };
   var a = { SinOsc.ar(400) };
   f.(a)
}
)
```

$$f(x) = 0.3x^2 - 0.8x^3$$
$$a = \sin(\frac{400}{2\pi}\mathring{t})$$
$$\mathring{y}(\mathring{t}) = f(a)$$

References

Galison, P. (2004). *Einstein's clocks, Poincaré's maps: empires of time*. WW Norton & Company.

Heidelberger, M. (2003). The Theory-Ladenness and Scientific Instruments in Experimentation. In Radder, H., editor, *The Philosophy of Scientific Experimentation*. University of Pittsburgh Press, Pittsburgh, PA, USA.

Hermann, T. and Ritter, H. (2004). Growing Neural Gas Sonification for Exploratory Analysis of High-dimensional Data. In *Proceedings of the International Symposium of Multimodal visualiztation*.

Kaper, H. G. and Tipei, S. (1999). Formalizing the concept of sound. In *Proceedings of International Computer Music Conference*, pages 387–390, Beijing, China.

Quine, W. V. O. (1951). Two dogmas of empiricism. *The Philosophical Review*, (60):20–43.

Xenakis, I. (2001). *Formalized Music: Thought and Mathematics in Composition (Harmonologia)*. Pendragon Press.